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April 10, 1996

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Dear Marc:

I would like thank you for your interest in our work and the support we received over the last three years. This has been a very interesting project not only from the academic point of view but also because it has been possible for us to implement some important results in Stress Check, and make them generally available for solving practical problems.

Enclosed are six copies of the final technical report. Should you have any questions, please call.

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13. ABSTRACT (*Maximum 200 words*)
This final technical report summarizes the main results achieved in this three-year project. Details are available in project reports and publications listed in this report.

The project was concerned with advancing the safety, durability, reliability and maintainability of DOD systems in the following four interrelated areas:

(a) Development and demonstration of advanced methods for modelling structural plates and shells made of laminated composites;
(b) Development and demonstration of improved methods for the design and analysis of fastened structural connections;
(c) Development of methods for the quantitative treatment of failure initiation events in metals and composites;
(d) Development of an iterative solver based on the "multi-p" method.

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ADVANCED MATHEMATICAL MODELS FOR STRUCTURAL SYSTEMS

Grant No. AFOSR F49620-93-1-0173

B. A. Szabó, Principal Investigator

Final Technical Report

Reporting period: 2/15/93-2/14/96.

Date of report: 10 April 1996.

1. SCOPE

This final technical report is submitted in compliance with instructions in Form AFSC 442, signed by Kathleen Wetherell, Contracting Officer, on February 12, 1993. The due date for this report is April 14, 1996.

The report summarizes the main results achieved in this project. Detailed technical descriptions are available in the related publications (see Section 7, Publications).

2. MAIN OBJECTIVES AND RESULTS

The main objective of this project was to investigate the feasibility of development of a software system, sufficiently advanced to make it possible to simulate structural and mechanical systems with approximately the same degree of reliability as if full-scale physical experiments had been performed. The specific focus of research covered the following four interrelated areas:

(a) Development and demonstration of advanced methods for modelling structural units fabricated of laminated composites;

(b) Development and demonstration of improved methods for the design and analysis of fastened structural connections;

(c) Development of methods for the quantitative treatment of failure initiation events in composite materials.

(d) Development of an iterative solver based on the "multi-p" method.

The most important accomplishments and new findings are in the area of composite materials technology which are summarized below.

A hierarchic family of models has been developed, tested and documented for structural plates made of laminated composites. This family of models makes it possible to perform structural and strength computations with a high degree of reliability and without having to change the discretization.

An advanced method has been developed for the analysis of failure initiation at multi-material interfaces. The method provides the eigenpairs which characterize the natural straining modes and the generalized stress intensity factors which characterize the energy available for causing failure. This technology is essential

for properly interpreting strength tests of composite materials.

These new capabilities have been made available to Air Force laboratories and contractors through a professional quality software called Stress Check.

3. ACCOMPLISHMENTS/NEW FINDINGS

3.1 Hierarchic Models for Laminated Plates

This research was concerned with the development of a hierarchic family of models for structural plates and shells fabricated from laminated composites. From the engineering point of view, two types of information are of interest: The structural response and the strength response. The structural response is typically characterized by stiffness coefficients, natural frequencies and buckling strength. The strength response is typically characterized by stress maxima, especially stresses at boundaries, and stress intensity factors. Designers are interested in the structural and strength responses in different phases of design. The hierarchic family of models developed under this project will make it possible to use the proper model for each application *without the need to change the finite element mesh.*

A test implementation has been completed, incorporated into the software infrastructure PEGASYS, and numerical experiments have been performed during the first year of the project. In the second year and third years the investigation focussed on the following areas:

1. Proper topological representation of structural shells in the context of the p-version of the finite element method was investigated. Recognizing that topological descriptions given by computer-aided design (CAD) systems are typically by non-uniform rational B-splines (NURBS), which cannot be incorporated directly into finite element analysis (FEA) codes, re-approximation of NURBS with piecewise polynomials was investigated. Of special importance is the fact that the piecewise polynomials possess C^0 continuity only, hence proper control of errors in stresses at the element interfaces, arising from lack of analyticity in the topological representation, was the principal goal of investigation. It was found that piecewise polynomial approximations, based on recently developed collocation points, yield very satisfactory results.

2. The resolution of boundary layers in the case of thin plates was investigated in collaboration with Professor Christoph Schwab of the University of Maryland. The existence of strong boundary layer effects in shear forces were demonstrated. This is especially important in the case of laminated plates. A surprising conclusion is that the bending moments do not exhibit similarly strong boundary layers. Furthermore, proper meshing for the recovery of shear forces depends on the hierarchic order.

3. In those areas where the plate is "thick", such as the neighborhoods of structural connections, it was found that the commonly used Reissner-Mindlin model is not reliable: Pointwise values of stresses depend strongly on the shear factors. This underlines the need for hierarchic models.

4. Work was started on the treatment of geometrically nonlinear problems and stability problems in the context of the p-version. The nonlinear problem was formulated, implemented and representative test cases solved for two classes of problems: (a) The class of problems which has an unique solution; and (b) the problem of elastic collapse, i.e., problems with turning points. A doctoral

dissertation has been completed.

3.2 Structural Connections

Fastened structural connections are common sites of failure initiation. Therefore it is important to use effective tools for the analysis and design of structural connections. In current professional practice fastened connections are often poorly treated, even conceptual errors are not uncommon.

Modeling is generally performed with two objectives in mind: To determine the distribution of forces in fastener groups and, having identified the most critical fasteners, to perform strength and durability analyses. This is possible only if the force-displacement relationships of fasteners and their dependence on installation, aging and manufacturing tolerances are represented by the model.

This project focussed on the following two areas:

1. The problem of analyzing the effectiveness of alternative cold-working processes. Cold-working involves the imposition of a sufficiently large radial displacement to cause plastic deformation in the vicinity of the fastener holes. The resulting residual stresses cause compression to occur around the perimeter of fastener holes, which increases their resistance to cracking. A method for accurate and reliable estimation of the residual stress zone in the context of the p-version was developed and tested.

2. The problem of modelling mechanical contact was investigated. In contact problems singularities arise at the boundaries of the contact zone. Treatment of contact by the hp-version is complicated because at each iteration a new mesh has to be created to capture the singular behavior. Recognizing the fact that the functional form of the singularities is known a priori, an alternative to remeshing is to enlarge the approximation space such that the essential characteristics of the solution in the vicinity of the fasteners are well represented by the approximation space. This work was concerned with the development of methods for handling the singular terms associated with contact. A doctoral dissertation has been completed.

3.3 Criteria for Failure Initiation

The principal objective of this work was to develop essential analytical procedures which will make it possible to correlate failure initiation events with mechanical and thermal loadings in composite materials. There are many applications such as, for example, electronic packaging, ceramics, adhesively bonded joints and laminated composites. The new method will replace expensive and unreliable trial-and-error methods which are currently used. Two separate procedures are performed:

First, it is necessary to compute by numerical means the eigenpairs which characterize the exact solution in the neighborhood of singular points. Numerical computation is necessary because the analytical expressions become extremely complicated in the case of anisotropic multi-material interfaces. Second, it is necessary to determine the amplitude of the eigenpairs in specific cases.

The eigenpairs are computed from the condition that they must satisfy the equations of anisotropic and inhomogeneous elasticity locally and satisfy the local boundary conditions. The formulation is based on a generalization of the "Steklov problem". The formulation is finite element-oriented and uses special features of

p-extensions. Numerical experiments have shown that the method performs very well. Several representative tests cases have been solved.

The amplitudes of the eigenpairs, called generalized stress intensity factors, are load-dependent. The eigenpairs and their coefficients characterize the natural straining modes at singular points. Therefore failure initiation criteria involve these characteristic data either directly or indirectly.

The generalized stress intensity factors are computed by a new method developed under this project. The new method is based on the complementary energy formulation. The major advantages of the new method are as follows:

1. The computed generalized stress intensity factors exhibit superconvergence.
2. The method is applicable to anisotropic materials, and any type of singularities.
3. The method can be used in conjunction with any finite element analysis program. A doctoral dissertation has been completed.

3.4 Iterative Solver

This research was concerned with a class of iterative methods for the solution of linear systems of equations arising from the p-version. The iterative method investigated, called the *multi-p method*, is analogous to the well-known multigrid methods, which are designed to solve systems of equations associated with finite difference or h-version of finite element methods.

A theory for *algebraic multi-p methods*, including the standard multi-p V-cycle method, nested multi-p method and accelerated multi-p V-cycle method, has been developed. These multi-p methods can be used as preconditioners for the conjugate gradient method. Applying the multi-p preconditioner, it was shown that the condition number of the stiffness matrix of p-version grows slower than $C(1+log^2 p)$. Numerical results indicate that the condition numbers of multi-p preconditioned systems are independent of p for practical purposes. Multi-p methods have been incorporated into the software infrastructure PEGASYS and have been demonstrated to be superior to direct solvers for some three-dimensional elasticity problems.

The convergence rate of an iterative method is strongly dependent on the condition number of the stiffness matrix. Therefore it is important to know what is the condition number of the p-version stiffness matrix. This information is available for the h-version. For the p-version, however, to our knowledge, there was no theory to characterize the condition number of the stiffness matrix. A general, yet simple approach has been developed to derive both the lower and upper bounds for the minimum eigenvalues, maximum eigenvalues and condition numbers of the p-version stiffness matrix. For a class of hierarchical basis functions used in the p-version, a complete characterization of eigenvalues of mass matrix and stiffness matrix have been developed.

4. PROJECT OVERVIEW

The main objectives of the project, as stated in the proposal are identified by bullets (•). The summaries of progress, results achieved, and pertinent remarks are identified by open circles (∘). There have been no changes in the goals identified in the project proposal.

- Develop advanced procedures for the numerical simulation of the mechanical action of fastener groups, utilizing space enrichment techniques. Emphasis is on reliable determination of force distribution among fasteners.
  - Implemented for thin plates (membrane models). Numerical tests have been conducted.
  - Investigation of contact between fastener pins and fastened plates has been undertaken. The goal is to determine stress maxima at the boundaries of the contact region.
  - The effects of plastic deformation, due to cold-working of fastener holes has been investigated. The algorithm is based in the deformation theory of plasticity.
- Implement the advanced procedures in the testbed PEGASYS.
  - Advanced constraint enforcement procedures have been implemented and tested.
  - A parallel solver based on the frontal solution method was developed. The solver is designed to utilize networked workstations for solving large systems of equations associated with the p-version. These equations are characterized by coarse granularity.
  - An iterative solver, based on the multi-p algorithm, has been developed and a test implementation completed.
  - Implemented and tested hierarchic models for structural plates.
- Perform benchmark studies using model problems which are representative of structural repairs of aircraft components.
  - Benchmark studies have been performed on fastener groups with multi-site damage.
  - Benchmark studies have been performed on cold-worked fasteners with the objective to determine residual stresses. Model problems of practical interest have been solved and the results discussed with engineers at McDonnell Douglas Aircraft Company.
- Develop adaptive procedures for the simulation of structural plates fabricated of laminated composites. Emphasis is on the achievement of effective control of both the discretization and modeling errors.
  - This work was in collaboration with Dr. Christoph Schwab of the University of Maryland. It was shown theoretically and by numerical means that it is possible to achieve *uniform exponential convergence* (i.e., independent of the plate/shell thickness) for the hp-version of the finite element method. Boundary layer effects have been demonstrated.
- Investigate and perfect procedures for obtaining reliable stopping criteria in model selection.
  - The number of degrees of freedom grows very rapidly as the model order is increased. Given that proper model selection depends on the engineering data of interest, and reliable error estimation techniques do not exist for functionals other than the energy norm, it is necessary to perform extensions with respect to model order. An extension of the iterative multi-p algorithm developed under this project appears to be the only practical

procedure for estimating the effects of increased hierarchic order on engineering data.

- Implement a baseline capability for adaptive models of laminated plates in the testbed PEGASYS.
  - This work has been completed.
- Perform benchmark studies representative of both structural and strength models. (See, for example, D.Sc. Dissertation by R. L. Actis, Washington University, 1991).
  - This work has been completed for structural plates.
- Extend the modeling capability to shells, including doubly curved shells. Investigate locking in the context of p-extensions.
  - An investigation of advanced mapping techniques for shells has been completed. It was shown that through the use of special collocation points accurate representation of generally curved shells is possible. A number of benchmark problems has been solved and documented.
- Select and solve model problems representative of the canopy of fighter aircraft. Coordinate model selection and simulation studies with Wright-Patterson AFB. Arrange for experimental correlation.
  - The Principal Investigator visited Wright Laboratory on March 6 and July 27, 1995. Dr. Max Blair and Mr. Greg Reich of the Flight Dynamics Directorate (WL/FIBGE) visited Washington University on April 21, 1995. A demonstration project was undertaken. At the close of this project parametric studies of an idealized integrally stiffened wing structure are continuing. Major Robert Canfield (AFIT/ENY) is the point of contact.
- Develop procedures for the determination of stress intensity factors for multi-material interfaces.
  - Determination of the eigenpairs which characterize temperature and stress fields (the natural straining modes and their thermal equivalents) based on the Steklov formulation has been completed. Papers have been written.
- Develop methods for the computation of stress maxima in complex geometries when it is impractical to model all fillets.
  - An algorithms has been developed and tested. The results have been presented to engineers at Ford Motor Company for comments.
- Implement, test, and demonstrate superconvergent extraction procedures for the computation of generalized stress intensity factors in the testbed PEGASYS.
  - This work has been completed. The results were demonstrated to visiting engineers from Rome Laboratory June 16, 1994. The Principal Investigator presented briefings at Rome Laboratory on February 18 and December 15, 1994.
- Apply superconvergent extraction procedures for the computation of generalized stress intensity factors to representative test cases (see, for example, report by D. A. Followell, S. L. Liguore and R. Perez, MDC B2303, McDonnell Aircraft Company, St. Louis, 1991).

- This work has been completed.

5. PERSONNEL SUPPORTED

5.1 Faculty:

Dr. Barna A. Szabó, PI, Professor of Mechanics

Dr. I. Norman Katz, Professor

5.2 Research Associates:

Mr. Kent Myers

Dr. Ricardo Actis

Dr. Xian Guo

5.3 Graduate Students:

Dr. Zohar Yosibash (Graduated in 1995)

Dr. Scott Prost-Domaski (Graduated in 1995)

Dr. Yehuda Volpert (Graduated in 1995)

Mr. Andre Noel (Graduation expected in 1996)

Mr. George Kiralyfalvi (Graduation expected in 1997)

6. PUBLICATIONS

Peer-reviewed publications submitted/and accepted.

[1] Yosibash, Z. and Szabó, B. "Numerical Analysis of Singular Points", in *Recent Developments in Computational Mechanics*, edited by P. K. Basu and A. Nagar, ASME, New York, AD-Vol. 39, pp. 29-44, (1993)

[2] Yosibash, Z. and Szabó, B. A., "Numerical Analysis of Singularities in Two Dimensions. Part 1: Computation of Eigenpairs", *International Journal for Numerical Methods in Engineering*, Vol. 38, pp. 2055-2082 (1995).

[3] Szabó, B. A. and Yosibash, Z., "Numerical Analysis of Singularities in Two Dimensions, Part 2: Computation of Generalized Flux/Stress Intensity Factors", *International Journal for Numerical Methods in Engineering*, Vol. 39, pp. 409-434 (1996).

[4] Szabó, B. A. and Yosibash, Z., "Superconvergent Computations of Flux Intensity Factors and First Derivatives by the FEM", to appear in: *Computer Methods in Applied Mechanics and Engineering*.

[5] Yosibash, Z. and Szabó, B. A., "Convergence of Stress Maxima in Finite Element Computations", *Communications in Applied Numerical Methods*, **10**, No. 9, pp. 683-697 (1994).

[6] Yosibash, Z. and Szabó, B. A., "Generalized Stress Intensity Factors in Linear Elastostatics", to appear in *International Journal of Fracture*.

[7] Yosibash, Z. and Szabó, B. A., "A Note on Numerically Computed Eigenfunctions and Generalized Stress Intensity Factors Associated with Singular

Points", Under review: *Engineering Fracture Mechanics*, submitted Nov. 94.

[8] Yosibash, Z., "On Solutions of Two-dimensional Linear Elastostatic and Heat-transfer Problems in the Vicinity of Singular Points", Under review: *International Journal of Solids and Structures*, submitted Jan. 95.

[9] Yosibash Z., "Numerical Thermo-Elastic Analysis of Singularities in Two-dimensions", Under review: *International Journal of Fracture*, submitted Apr. 95.

[10] Szabó, B. A. and Actis, R. L., "The Finite Element Method in Professional Practice", to appear in: *Computer Methods in Applied Mechanics and Engineering.*

[11] Szabó, B. A., Actis, R. L. and Holzer, S. M., "Solution of Elastic-Plastic Stress Analysis Problems by the p-Version of the Finite Element Method", *Modeling, Mesh Generation and Adaptive Numerical Methods for Partial Differential Equations*, Edited by J. E. Flaherty et. al., IMA Volumes in Mathematics and its Applications (to appear).

[12] Guo, X. Z. and Myers, K. W., "A Parallel Solver for the HP-Version of Finite Element Methods", to appear in *Computational Methods in Applied Mechanics and Engineering.*

[13] Hu, N., Guo, X. Z. and Katz, I. N., "Algebraic Multi-p Methods and Multi-p Preconditioners" To appear in the *SIAM Journal on Scientific Computing.*

[14] Hu, N., Guo, X. Z. and Katz, I. N., "Lower and Upper Bounds for Eigenvalues and Condition Numbers in the p-Version of Finite Element Method", Submitted for publication in the *SIAM Journal on Scientific Computing.*

[15] Noel, A. T. and Szabó, B. A., "Solution of Geometrically Nonlinear Problems Using Spatial Reference and the p-Version of the FEM", Proc., 1996 ASME International Computers in Engineering Conference, Irvine, CA, Aug. 19-22, 1996.

7. INTERACTIONS/TRANSITIONS

7.1 Participation/presentations at meetings, conferences, seminars:

[1] Yosibash, Z., "Numerical Thermo-elastic Analysis of Singularities in Two-dimensions", Finite Element Circus Meeting, Mar. 24-25, 1995, Brookhaven National Laboratory, Long Island, USA.

[2] Yosibash, Z., "Numerical Analysis of Singularities Associated with the Mechanical and Thermal Problems of Electronic Packaging", Austin, Texas, USA. Symposium on Computational and Applied Mathematics I, Austin, Texas, Apr. 20-22, (1995).

[3] Yosibash, Z. and Szabó, B. "Superconvergent Extraction of Stress Intensity Factors and Stresses from Finite Element Solutions" Third U.S. National Congress on Computational Mechanics, Dallas, TX June 11-13, 1995.

[3] Szabó, B. A., "Finite Element Analysis in Professional Practice", Ford Motor Company, Finite Element Users' Group, March 10, 1995.

[4] Szabó, B. A., "Finite Element Analysis in Professional Practice", Seminar on Quality of Scientific and Technical Computing, Seriate, Italy, April 11-12 1995.

[4] Szabó, B. A., "Hierarchic Models for Laminated Composites", International Congress on Industrial and Applied Mathematics (ICIAM), Hamburg, Germany, July 3-7, 1995.

[5] Szabó, B. A., "The p-Version of the Finite Element Method in Professional Practice", International Conference on Computational and Engineering Science, Mauna Lani, Hawaii July 30-Aug. 3, 1995.

7.2 Consultative and advisory functions to Air Force laboratories and contractors:

[1] Szabó, B. A., "Numerical Analysis of Singular Points" Seminar at Rome Laboratory, February 18, 1994.

[2] Szabó, B. A., "New Developments in Finite Element Analysis", Seminar at Lockheed/Fort Worth Company, April 11, 1994.

[3] Szabó, B. A., "Developments in Simulation Software", Seminar at Sandia National Laboratories, June 1, 1994.

[4] Szabó, B. A., "Numerical Analysis of Material Interface Singularities in Two Dimensions", AFOSR Contractors/Grantees Meeting in Computational and Physical Mathematics, Phillips Laboratory, Kirtland AFB, June 2, 1994.

[5] Szabó, B. A., "New Developments in Finite Element Analysis", Seminar at McDonnell Aircraft Company, July 11, 1994.

[6] Yosibash, Z. and Actis, R. L., "Advanced Methods for the Computation of Stress Intensity Factors", Poster presentation, USAF Aircraft Structural Integrity Program Conference, Dec. 6-8, 1994, San Antonio, Texas, USA.

[7] Szabó, B. A., "The Software Infrastructure PEGASYS", Seminar at Rome Laboratory, December 15, 1994.

[8] Szabó, B. A., "Finite Element Analysis in Professional Practice", Wright Laboratory, March 6, 1995

[9] Szabó, B. A., "Advanced Mathematical Models for Structural Plates and Shells", AFOSR Contractors/Grantees Meeting in Computational and Physical Mathematics, Phillips Laboratory, Kirtland AFB, June 29, 1995.

[10] Szabó, B. A., "Advanced Surface Representation in Finite Element Analysis", Wright Lab., Wright-Patterson AFB, July 27, 1995.

[11] The principal investigator is maintaining contact with Dr. Max Blair (WL/FIBGE) and his associate Dr. Robert Canfield, Major, USAF (AFIT/ENY) on the development of procedures for advanced design of military aircraft. The contacts are through personal visits, e-mail and telephone.

7.3 Transitions.

The new technological capabilities, such as hierarchic models for laminated plates and computation of generalized stress intensity factors, have been made available through a professional quality software product, called Stress Check. The users of Stress Check include Rome Laboratory, McDonnell Douglas Aerospace Company, Rockwell International, NASA Johnson Space Center, Ford Motor Company and others. Stress Check is being developed with private capital.